



Is Deuteron a Six Quark System?

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ABSTRACT

The "deuteron-like" six quark spherical bag is discussed and compared with the real deuteron. Mixtures of di-baryon components with hidden color are exhibited. A tunneling transition of deuteron into the six quark bag with probability about 7×10^{-2} is argued.

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The intriguing question on whether quark degrees of freedom play a noticeable role in understanding nuclear events has provoked in the last years a number of interesting works in that direction.^{1, 2}

The recent measurements of electromagnetic form factor of deuteron at large transferred momenta³ as well as the indications on existence of the cumulative effects in relativistic nuclear collisions⁴ have poured new enthusiasm into attempts to treat nucleus as a system of quarks rather than nucleons.⁵⁻¹¹ Due to the lack of consistent theory of quark confinement most of calculations in that field are made in the framework of the MIT-quark-Bag model.^{12, 13} In a recent paper by R. L. Jaffe¹⁴ this model was used in discussion of regularities of a spectrum of the di-baryonic systems composed of six quarks confined to a static spherically symmetric volume.

In this note we present some new information on the structure of such di-baryonic quark states and discuss their possible interpretation. We consider mostly the six quark system with the quantum numbers of deuteron to which we shall refer hereafter as a "d-like" quark bag. Obviously, the complete wave function of a system of six quarks all having angular momentum $j = 1/2$ as follows from calculations in the MIT-Bag model, must be antisymmetric with respect to the spin, isospin and color, i. e.

$$\psi(6q) = \frac{1}{\sqrt{20}} \left(1 - \sum_{\substack{i=1,2,3 \\ j=4,5,6}} P_{ij} \right) \left(\begin{array}{c} \text{color} \\ \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array} \times \begin{array}{c} \text{spin-isospin} \\ \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \end{array} \right), \quad (1)$$

the colorless component of the wave function belongs to the 50-plet of the $SU(4)^{SI}$ symmetry group with the spin-isospin content given by¹⁵

$$50 = (3, 1) + (1, 3) + (5, 3) + (3, 5) + (7, 1) + (1, 7). \quad (2)$$

The first term is the r. h. s of Eq. (2) has quantum numbers of deuteron, the second one of the virtual singlet state known is the phenomenology of low energy nucleon-nucleon scattering, etc.¹⁶

In the following we shall use knowledge of an explicit form of the wave function of a six quark system to answer an interesting and crucial for physical interpretation question on the di-baryon composition of the state. We shall show, particularly, that "d-like" quark bag is composed almost equally from two nucleons and two Δ -isobars and contains also a considerable mixture of the di-baryon component with "hidden" color.

The relative couplings of the "d-like" quark bag to the pn- and $\Delta\Delta$ -systems are determined by the SU_4 Clebsch-Gordan coefficients for the decomposition of the product $20 \times 20 = 50 + \dots$. Since most of relevant calculations in this field¹⁷ are done by using the reduction chain $SU(4) \supset SU(3)$ we give the representations of the state vectors of the d-like quark by:

$$\begin{aligned} |d_{J_z=+1}\rangle = & \sqrt{\frac{2}{15}} |15; \frac{3}{2}, \frac{3}{2}\rangle - \frac{\sqrt{5}}{3} |\overline{15}; \frac{1}{2}, \frac{1}{2}\rangle - \\ & - \frac{2\sqrt{2}}{3\sqrt{5}} |\overline{15}; \frac{3}{2}, \frac{1}{2}\rangle + \sqrt{\frac{2}{15}} |\overline{10}; \frac{3}{2}, -\frac{1}{2}\rangle, \end{aligned} \quad (3)$$

of the nucleons:

$$\begin{aligned} |p_{1/2}\rangle = & \sqrt{\frac{2}{3}} |6; 1, 1\rangle - \frac{1}{\sqrt{3}} |10; 1, 0\rangle \\ |n_{1/2}\rangle = & \sqrt{\frac{2}{3}} |10; \frac{1}{2}, -\frac{1}{2}\rangle + \frac{1}{\sqrt{3}} |6; \frac{1}{2}, \frac{1}{2}\rangle, \end{aligned} \quad (4)$$

and of the isobars:

$$Q = 2 \quad |\Delta_{J_z}^{++} \rangle = |10; \frac{3}{2}, J_z \rangle \quad J_z = \frac{3}{2}, \pm \frac{1}{2}$$

$$Q = 1 \quad |\Delta_{3/2}^+ \rangle = |10; 1, 1 \rangle$$

$$|\Delta_{1/2}^+ \rangle = \sqrt{\frac{2}{3}} |10; 1, 0 \rangle + \frac{1}{\sqrt{3}} |6; 1, 1 \rangle$$

$$|\Delta_{-1/2}^+ \rangle = \sqrt{\frac{2}{3}} |6; 1, 0 \rangle + \frac{1}{\sqrt{3}} |10; 1, -1 \rangle$$

$$Q = -1 \quad |\Delta_{3/2}^- \rangle = |10; 0, 0 \rangle \tag{5}$$

$$|\Delta_{1/2}^- \rangle = |6; 0, 0 \rangle$$

$$|\Delta_{-1/2}^- \rangle = |3; 0, 0 \rangle$$

$$Q = 0 \quad |\Delta_{3/2}^0 \rangle = |10; \frac{1}{2}, \frac{1}{2} \rangle$$

$$|\Delta_{1/2}^0 \rangle = \sqrt{\frac{2}{3}} |6; \frac{1}{2}, \frac{1}{2} \rangle + \frac{1}{\sqrt{3}} |10; \frac{1}{2}, -\frac{1}{2} \rangle$$

$$|\Delta_{-1/2}^0 \rangle = \sqrt{\frac{2}{3}} |6; \frac{1}{2}, -\frac{1}{2} \rangle + \frac{1}{\sqrt{3}} |3; \frac{1}{2}, \frac{1}{2} \rangle .$$

We recall the reduction with respect to SU(3) of the SU(4) irreducible representations of dimensions 50, which contains the "d-like" states, and 20 (symmetric) to which belongs the nucleons and the above Δ :

$$50 = 10 + 15 + \overline{15} + \overline{10} . \tag{6}$$

$$20 = 10 + 6 + 3 + 1 . \tag{7}$$

Using the Eqs. (4) & (5) and the table values of the SU(4)-Clebsch-Gordan coefficients we find all the couplings in question (see Table I).

TABLE I. Couplings of d-like quark bag with $J_z = 1$ to the different di-baryonic systems.

Spins Charges	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{2}, -\frac{1}{2}$	$-\frac{1}{2}, \frac{3}{2}$
$\Delta^{++}\Delta^{-}$	$\frac{2}{3\sqrt{10}}$	$-\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{30}}$
$\Delta^{+}\Delta^0$	$-\frac{2}{3\sqrt{10}}$	$\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{30}}$
pn	$\frac{5}{3\sqrt{10}}$	----	----

As one can see from the Table I the relative probability of the isobar and nucleon components with the same charges and spin projections, i. e.

$$\left| \frac{\Delta_{1/2}^{+} \Delta_{1/2}^0}{p_{1/2} n_{1/2}} \right|^2 = \left(\frac{2}{5} \right)^2 = 16\%, \quad (8)$$

is too large as compared with what could be expected in the theory of nuclear matter.¹⁸

Moreover an overall composition of the "d-like" quark bag is obtained taking into account the complete antisymmetry of the wavefunction. This can be done¹⁹ using the representations: $220 = (1^3)$ and $924 = (1^6)$ -- to which belong the nucleons and the "d-like" quark bag respectively -- of the group $SU(12) \supset SU(4)^{SI} \times SU(3)^{color}$ -- the results are as follows:

proton-neutron: $10/90 \approx 11\%$

isobar component: $8/90 \approx 9\%$

baryon pairs with "hidden color": $\underline{72/90 = 80\%}$

Total = 100%

The component with a "hidden color" corresponds to splitting of the 50-plet onto the product of different couples of SU_4 multi-plets $20'$, $20''$ and 4 , describing states with the unit baryonic charge and nontrivial color. Thus, the "d-like" quark bag is built up to 80% off the color baryon pairs contrary to the real deuteron which is predominately a loosely bound proton-neutron system.

We would like to emphasize that this result is based uniquely on the fact that all six quarks are sitting on the same energy shell with $j = 1/2$. A more realistic quark model of deuteron may be obtained by considering a space and time dependent shape of the bag boundary and hence a more rich spectrum of quark excitations including higher values of angular momentum of quarks.⁷

Instead, we shall consider here a possibility that the d-like quark bag gives reasonable description of the real deuteron in the processes at large transferred momenta. Gross features of these processes governed by the short-distance dynamics have not to be affected presumably by such long-range details as a real shape or a size of the bag.

The recent measurements of the electromagnetic form factor of deuteron at large transferred momentum $0.8 \leq q^2 \leq 6.0 \text{ GeV}/c^2$ gaining the result $F_d(q^2) \sim (1/q^2)^{5.0 \pm 0.5}$,³ have shown the remarkable agreement with the prediction of the dimensional quark counting for the exponent $n_d - 1 = 5$, where n_d is a minimal number of elementary constituent of deuteron.^{5,6}

At the same time it was found that direct analytic extrapolation of those data to the point $q^2 = 0$ yields a value which is by one order of magnitude lower of what that could be expected from the total deuteron charge normalization $F_d(0) = 1$. For instance, "the pentapole" fit²⁰ $F_d(q^2) = A(1 - aq^2)^{-5}$ with $a = 0.51 \text{ GeV}/c^{-2}$ gives $F_d(0) = A = 7 \times 10^{-2}$. We interpret this result as an indication on possibility of deuteron transition into the d-like quark bag. The transition rate can be estimated as a probability of tunneling through the "hard-core" potential of deuteron into the configuration where the proton and nucleon centers of masses are coincide, i. e.²¹

$$\alpha \sim e^{-\frac{2}{\hbar} \text{Jm} \int_0^a dr \sqrt{-2m_d(\nu_0 + \epsilon_d)}}. \quad (14)$$

Here ν_0 is a height of the internuclear hard-core potential with an effective size a ; $m_d = m_p m_n / (m_p + m_n)$ and ϵ_d are respectively the reduced mass and the binding energy of the deuteron.

It was suggested that the height of the hard-core potential is determined by an excess of the classical energy of the quark bag above two nucleon masses, i. e.

$$\nu_0 = E(\text{d-like bag}) - 2m_p \approx 0.27 \text{ GeV}. \quad (15)$$

Taking values of the effective radius of the hard-core potential a in the range²² (0.4 – 0.6) fm one gets from Eq. (14) (13 – 5)% as a crude estimation of the transition rate; in a remarkable agreement with the value suggested by the experimental data.

Going along this line we can estimate the net composition of a real deuteron as

$$pn - \text{component} \approx 93.8\%$$

$$\Delta\Delta - \text{component} \approx 0.6\%$$

$$\text{"hidden color"} \approx 5.6\%$$

Thus, due to the fact, that there exists appreciable admixture of the di-baryonic component with the hidden color in the real deuteron, no theory of the nuclear matter can give apparently an exhaustive description of nucleus without referring to the notion of colored quarks and gluons.

Search of direct experimental indications on existence of the hidden color component of nuclear matter would be an interesting task of great importance.

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Note that from our checking of the corresponding Table I, the state $J = 3$

in the 175 representation of the color-spin SU(6) group is missing; this state would carry a mass of 2357 MeV.

¹⁵For the sake of simplicity, we limit ourselves to the non strange states only. In terms of the SU(6) spin-flavor symmetry we have to consider the 490-plet. This is in accordance with an assignment of the deuteron to the decouplet of SU(3), suggested by R. J. Oakes, Phys. Rev. 131, 2239 (1963), and has been discussed also by F. J. Dyson and N. H. Xung, Phys. Rev. Lett. 13, 815 (1964).

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